5.1 Modelling Polynomials

FOCUS

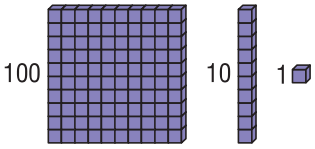
- Model, write, and classify polynomials.

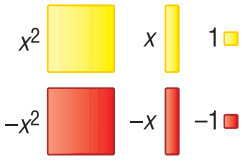
In arithmetic, we use Base Ten Blocks to model whole numbers. How would you model the number 234?

In algebra, we use algebra tiles to model integers and variables.

Yellow represents positive tiles. Red represents negative tiles.

How are Base Ten Blocks and algebra tiles alike?





Investigate

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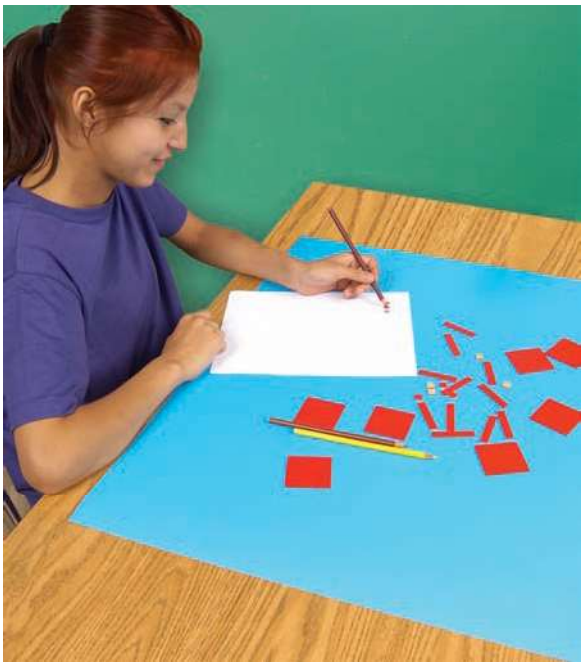


Use algebra tiles.

➤ Model each expression. Sketch the tiles.
How do you know which tiles to use?
How do you know how many of each tile to use?

- $x^2 + x - 3$
- $-2x^2 - 3$
- $2x^2 + 3x$
- $-2x^2 - 3x + 1$
- $-3x + 3$

➤ Write your own expression.
Have your partner model it with tiles.
Model your partner's expression with tiles.

Reflect & Share

For the first activity, compare your sketches with those of another pair of students.
Did you use the same tiles each time? If not, is one of you wrong?
Could both of you be correct? Explain.
Did the order in which you laid out the tiles matter? Explain.

Connect

We can use algebra tiles to model an expression such as $3x^2 - 2x + 5$.

To model $3x^2 - 2x + 5$, we use three x^2-tiles, two $-x$-tiles, and five 1-tiles.

A **polynomial** is one term or the sum of terms whose variables have whole-number exponents.

The expression $3x^2 - 2x + 5 = 3x^2 + (-2)x + 5$ is an example of a polynomial in the variable x.
This polynomial has 3 terms: $3x^2$, $(-2)x$, and 5
Terms are numbers, variables, or the product of numbers and variables.
The **coefficients** of the variable are 3 and -2.

The term with the greatest exponent determines the **degree** of the polynomial. This polynomial has degree 2.

$3x^2 - 2x + 5$

The term 5 is a **constant term**. Its value does not change when the value of x changes. A constant term has degree 0.

The term $-2x$ has degree 1 because $-2x = -2x^1$.

We can use any variable to write a polynomial and to describe the tiles that model it.
For example, the tiles that model the polynomial $-5n^2 + 7n - 1$ also model the polynomial $-5p^2 + 7p - 1$.

We can also classify a polynomial by the number of terms it has.
Polynomials with 1, 2, or 3 terms have special names.
A **monomial** has 1 term; for example: $4a$, 6, $-2p^2$
A **binomial** has 2 terms; for example: $2c - 5$, $2m^2 + 3m$
A **trinomial** has 3 terms; for example: $2h^2 - 6h + 4$

A polynomial is usually written in descending order; that is,
the exponents of the variable decrease from left to right;
for example, the polynomial $2k - 4k^2 + 7$ is written as $-4k^2 + 2k + 7$.

An algebraic expression that contains a term with a variable in the denominator, such as $\frac{3}{n}$, or the square root of a variable, such as $\sqrt{n}$, is *not* a polynomial.

Example 1 Recognizing the Same Polynomials in Different Variables

Which of these polynomials can be represented by the same algebra tiles?

a) $3x^2 - 5x + 6$ b) $-5 + 6r + 3r^2$ c) $-5m + 6 + 3m^2$

Justify the answer.

A Solution

a) $3x^2 - 5x + 6$

Use three x^2-tiles, five $-x$-tiles, and six 1-tiles.

b) $-5 + 6r + 3r^2$

Use five -1-tiles, six r-tiles, and three r^2-tiles.

c) $-5m + 6 + 3m^2$

Use five $-m$-tiles, six 1-tiles, and three m^2-tiles.

In parts a and c, the same algebra tiles are used.

So, the polynomials $3x^2 - 5x + 6$ and $-5m + 6 + 3m^2$ can be represented by the same tiles.

Example 2 Modelling Polynomials with Algebra Tiles

Use algebra tiles to model each polynomial.

Is the polynomial a monomial, binomial, or trinomial? Explain.

a) $-2x^2$ b) $2b^2 - b + 4$ c) $5a - 3$

A Solution

a) To represent $-2x^2$, use two $-x^2$-tiles.

Since there is only one type of tile, $-2x^2$ is a monomial.

b) To represent $2b^2 - b + 4$, use two b^2-tiles, one $-b$-tile, and four 1-tiles.

Since there are 3 types of tiles, $2b^2 - b + 4$ is a trinomial.

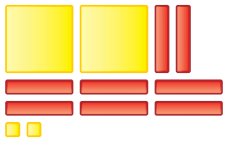
c) To represent $5a - 3$, use five a-tiles and three -1-tiles. Since there are 2 types of tiles, $5a - 3$ is a binomial.

Two polynomials are *equivalent* when they can be represented by identical algebra tiles.

Example 3 Recognizing Equivalent Polynomials

a) Which polynomial does each group of algebra tiles represent?

Model A



Model B

Model C

b) Which of the polynomials in part a are equivalent? How do you know?

A Solution

a) Use a table.

Model	Description of Tiles	Polynomial
A	two x^2-tiles, eight $-x$-tiles, and two 1-tiles	$2x^2 - 8x + 2$
B	eight $-x$-tiles, two x^2-tiles, and two 1-tiles	$-8x + 2x^2 + 2$
C	four $-x$-tiles and six 1-tiles	$-4x + 6$

b) Both models A and B contain the same tiles. The polynomials represented by these tiles have the same degree, and the same terms: $2x^2$, $-8x$, and 2
Both polynomials can be written as: $2x^2 - 8x + 2$
So, $2x^2 - 8x + 2$ and $-8x + 2x^2 + 2$ are equivalent polynomials.
Model C has no x^2-tiles, so its degree is different from that of models A and B.

Discuss the ideas

1. In the polynomial $3 + 2p$, which term is the constant term? How are constant terms modelled with algebra tiles?

2. Suppose you are given an algebra tile model of a polynomial. How can you identify the terms, the coefficients, and the degree of the polynomial? How can you identify the constant term?

3. What do we mean by "equivalent polynomials"? How can you determine whether two polynomials are equivalent?

Practice

Check

4. Which of the following expressions are polynomials? Explain how you know.

a) $2 + 3n$
b) $3\sqrt{x}$
c) $-5m + 1 + 2m^2$
d) 7
e) $\frac{1}{x^2} + \frac{1}{x} + 1$
f) $\frac{1}{2}s$

5. Is each expression a monomial, binomial, or trinomial? Explain how you know.

a) $3t + 4t^2 - 2$
b) $5 - 3g$
c) $9k$
d) 11

6. Name the coefficient, variable, and degree of each monomial.

a) $-7x$
b) $14a^2$
c) m
d) 12

7. Identify the degree of each polynomial. Justify your answers.

a) $7j^2 + 4$
b) $9x$
c) $2 - 5p + p^2$
d) -10

Apply

8. Identify the polynomials that can be represented by the same set of algebra tiles.

a) $x^2 + 3x - 4$
b) $-3 + 4n - n^2$
c) $4m - 3 + m^2$
d) $-4 + r^2 + 3r$
e) $-3m^2 + 4m - 3$
f) $-h^2 - 3 + 4h$

9. Name the coefficients, variable, and degree of each polynomial. Identify the constant term if there is one.

a) $5x^2 - 6x + 2$
b) $7b - 8$
c) $12c^2 + 2$
d) $12m$
e) 18
f) $3 + 5x^2 - 8x$

10. One student says, "$4a$ is a monomial." Another student says, "$4a$ is a polynomial." Who is correct? Explain.

11. Use algebra tiles to model each polynomial. Sketch the tiles.

a) $4x - 3$
b) $-3n - 1$
c) $2m^2 + m + 2$
d) $-7y$
e) $-d^2 - 4$
f) 3

12. Match each polynomial with its corresponding algebra tile model.

a) $r^2 - r + 3$
b) $-t^2 - 3$
c) $-2v$
d) $2w + 2$
e) $2s^2 - 2s + 1$

Model A

Model B

Model C

Model D

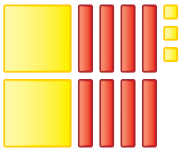
Model E

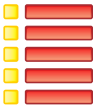
13. Which polynomial does each collection of algebra tiles represent?
Is the polynomial a monomial, binomial, or trinomial? Explain.

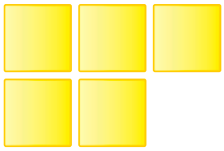
a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

14. Write a polynomial with the given degree and number of terms. Use algebra tiles to model the polynomial. Sketch the tiles.

a) degree 1, with 2 terms

b) degree 0, with 1 term

c) degree 2, with 1 term

d) degree 2, with 3 terms and constant term 5

15. Identify which polynomials are equivalent.
Explain how you know.

a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

i) 

16. Identify which polynomials are equivalent.
Justify your answers.

a) $5 - v + 7v^2$

b) $7v + 5 - v^2$

c) $5v + v^2 - 7$

d) $-7 + 5v + v^2$

e) $5 - v^2 + 7v$

f) $7v^2 + v + 5$

17. Write an expression that is *not* a polynomial. Explain why it is not a polynomial.

18. Assessment Focus

a) Use algebra tiles to model each polynomial. Sketch the tiles. Identify the variable, degree, number of terms, and coefficients.
   i) $-2x - 3x^2 + 4$
   ii) $m^2 + m$

b) Write a polynomial that matches this description:
a polynomial in variable c, degree 2, binomial, constant term -5

c) Write another polynomial that is equivalent to the polynomial you wrote in part b. Explain how you know that the polynomials are equivalent.

19. a) Write as many polynomials as you can that are equivalent to $-8d^2 - 3d - 4$. How do you know you have written all possible polynomials?

b) Which polynomial in part a is in descending order? Why is it useful to write a polynomial in this form?

Take It Further

20. The *stopping distance* of a car is the distance the car travels between the time the driver applies the brakes and the time the car stops. The polynomial $0.4s + 0.02s^2$ can be used to calculate the stopping distance in metres of a car travelling at s kilometres per hour on dry pavement.

a) Determine the stopping distance for each speed:
   i) 25 km/h ii) 50 km/h iii) 100 km/h

b) Does doubling the speed double the stopping distance? Explain.

Reflect

What is a polynomial?
How can you represent a polynomial with algebra tiles and with symbols?
Include examples in your explanation.

Math Link

Your World

A polynomial can be used to model projectile motion. When a golf ball is hit with a golf club, the distance the ball travels in metres, in terms of the time t seconds that it is in the air, may be modelled by the polynomial $-4.9t^2 + 22.8t$.