5.2 Like Terms and Unlike Terms

FOCUS

- Simplify polynomials by combining like terms.

When you work with integers,
a 1-tile and a −1-tile form a zero pair. → 0

What do you think happens when you combine algebra tiles with opposite signs?
Which expression do these tiles represent?

Investigate

2

You will need algebra tiles and a paper bag.

➤ Put both colours of algebra tiles in a bag.
Take a handful of tiles and sketch them.
Construct a table to record your work.

Algebra Tile Model	Symbolic Record

Use symbols to write the polynomial modelled by the tiles. Remove zero pairs.
Sketch the tiles that remain.
Use symbols to write the polynomial represented by the smaller set of tiles.

➤ Return the algebra tiles to the bag.
Repeat the activity 4 more times.

Reflect & Share

Share your results with another pair of students.
How could you verify each other's results?
When can you remove zero pairs from a set of tiles?
How does removing zero pairs help you simplify the polynomial that represents the set of tiles?

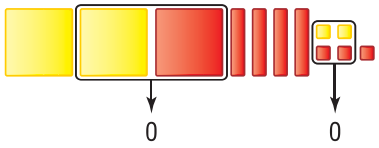
Connect

Here is a collection of red and yellow algebra tiles:

We organize the tiles by grouping like tiles:

These tiles represent the polynomial: $2x^2 - x^2 - 4x + 2 - 3$

We simplify the tile model by removing zero pairs.

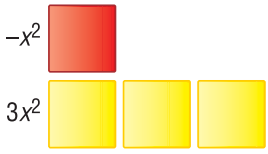


The remaining tiles represent the polynomial: $x^2 - 4x - 1$
We say that the polynomial $2x^2 - x^2 - 4x + 2 - 3$ *simplifies* to $x^2 - 4x - 1$.

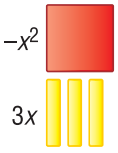
A polynomial is in *simplified form* when:
- its algebra tile model uses the fewest tiles possible
- its symbolic form contains only one term of each degree and no terms with a zero coefficient

Terms that can be represented by algebra tiles with the same size and shape are called **like terms**.

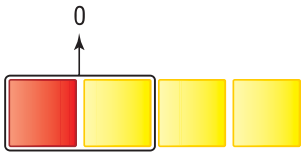
$-x^2$ and $3x^2$ are like terms.
Each term is modelled with x^2-tiles.
Each term has the same variable, x,
raised to the same exponent, 2.



$-x^2$ and $3x$ are *unlike terms*.
Each term is modelled with a different algebra tile.
Each term has the variable x,
but the exponents are different.



To simplify a polynomial, we group like terms and remove zero pairs.
$-x^2 + 3x^2$ simplifies to $2x^2$.



We can also simplify a polynomial by adding the coefficients of like terms. This is called *combining like terms.*

$-x^2 + 3x^2 = -1x^2 + 3x^2$ Add the integer coefficients: $-1 + 3 = 2$
$= 2x^2$

The polynomials $-x^2 + 3x^2$ and $2x^2$ are *equivalent.*
So, a polynomial in simplified form is also the equivalent polynomial in which all the like terms have been combined.

$-x^2 + 3x$ cannot be simplified.
We may not add coefficients when we have unlike terms.

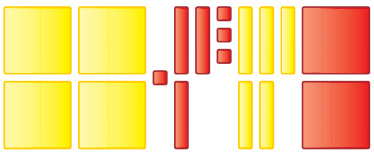
Example 1 Using Algebra Tiles to Simplify a Polynomial

Use algebra tiles to simplify the polynomial $4n^2 - 1 - 3n - 3 + 5n - 2n^2$.
Record the process symbolically.

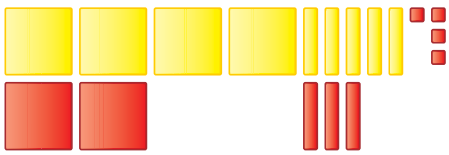
A Solution

Tile Model

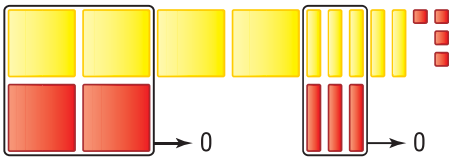
Display $4n^2 - 1 - 3n - 3 + 5n - 2n^2$.



Group like tiles.



Remove zero pairs.



The remaining tiles represent $2n^2 + 2n - 4$.

Symbolic Record

$4n^2 - 1 - 3n - 3 + 5n - 2n^2$

Group like terms:

$4n^2 - 2n^2 + 5n - 3n - 1 - 3$

Combine like terms:

$2n^2 + 2n - 4$

Example 2 Simplifying a Polynomial Symbolically

Simplify: $14x^2 - 11 + 30x + 3 + 15x - 25x^2$

A Solution

We need many tiles to model this polynomial.
So, we simplify it symbolically.

$14x^2 - 11 + 30x + 3 + 15x - 25x^2$ Group like terms.
$= 14x^2 - 25x^2 + 30x + 15x - 11 + 3$ Add the coefficients of like terms.
$= -11x^2 + 45x - 8$

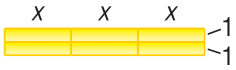
In *Example* 2, the polynomials $14x^2 - 11 + 30x + 3 + 15x - 25x^2$ and $-11x^2 + 45x - 8$ are equivalent.
Polynomials can be used to represent measures such as the side lengths of shapes.

Example 3 Investigating Situations that Represent Polynomials

a) Write a polynomial to represent the perimeter of each rectangle.

i)

ii)

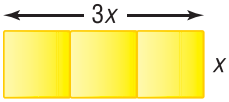


b) Each polynomial represents the perimeter of a rectangle.
Use algebra tiles to make the rectangle.

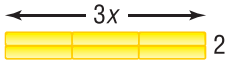
i) $4a + 2$ **ii)** $10b$

A Solution

a) i) The dimensions of the rectangle are $3x$ and x. So, the perimeter of the rectangle is:
$3x + x + 3x + x = 8x$



ii) The dimensions of the rectangle are $3x$ and 2. So, the perimeter of the rectangle is:
$3x + 2 + 3x + 2 = 6x + 4$

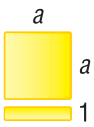


b) i) The perimeter is $4a + 2$.
Work backward.
Write the polynomial as the sum of equal pairs of terms.
$4a + 2 = 2a + 2a + 1 + 1$
The dimensions of the rectangle could be $2a$ and 1.

Another solution is:

$4a + 2 = a + (a + 1) + a + (a + 1)$

The dimensions of the rectangle could be a and $a + 1$.

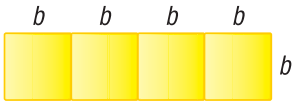


ii) The perimeter is $10b$.

Write the polynomial as the sum of equal pairs of terms.

$10b = 4b + 4b + b + b$

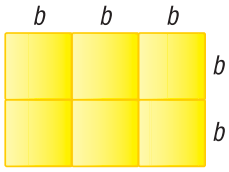
The dimensions of the rectangle could be $4b$ and b.



Another solution is:

$10b = 3b + 3b + 2b + 2b$

The dimensions of the rectangle could be $3b$ and $2b$.



A polynomial may contain more than one variable. Here is a polynomial in x and y:

$-2x^2 + 3xy + y^2 - 4x - 8y$

Example 4 Simplifying a Polynomial in Two Variables

Simplify: $4xy - y^2 - 3x^2 + 2xy - x - 3y^2$

A Solution

$4xy - y^2 - 3x^2 + 2xy - x - 3y^2$ Group like terms.

$= 4xy + 2xy - y^2 - 3y^2 - 3x^2 - x$ Combine like terms.

$= 6xy - 4y^2 - 3x^2 - x$

Discuss the ideas

1. Why can we combine like terms? Why can we not combine unlike terms?

2. How can you identify and combine like terms in an algebra tile model?

3. How can you identify and combine like terms symbolically?

Practice

Check

4. a) Use algebra tiles to model $3d$ and $-5d$. Sketch the tiles.

b) Are $3d$ and $-5d$ like terms? How can you tell from the tiles? How can you tell from the monomials?

5. a) Use algebra tiles to model $4p$ and $2p^2$. Sketch the tiles.

b) Are $4p$ and $2p^2$ like terms? How can you tell from the tiles? How can you tell from the monomials?

Apply

6. From the list, which terms are like $8x$?
$-3x$, $5x^2$, 4, $3x$, 9, $-11x^2$, $7x$, -3
Explain how you know they are like terms.

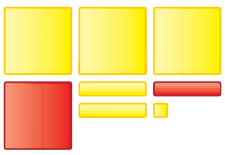
7. From the list, which terms are like $-2n^2$?
$3n$, $-n^2$, -2, $4n$, $2n^2$, -2, 3, $5n^2$
Explain how you know they are like terms.

8. For each part, combine tiles that represent like terms.
Write the simplified polynomial.

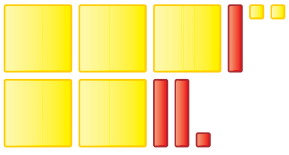
a)

b)

c)



d)



e)

f)

9. Identify the equivalent polynomials in the diagrams below. Justify your answers.

a)

b)

c)

d)

e)

f)

10. A student made these mistakes on a test.

- The student simplified $2x + 3x$ as $5x^2$.
- The student simplified $4 + 3x$ as $7x$.

Use algebra tiles to explain what the student did wrong.
What are the correct answers?

11. Use algebra tiles to model each polynomial, then combine like terms. Sketch the tiles.

a) $2c + 3 + 3c + 1$

b) $2x^2 + 3x - 5x$

c) $3f^2 + 3 - 6f^2 - 2$

d) $3b^2 - 2b + 5b + 4b^2 + 1$

e) $5t - 4 - 2t^2 + 3 + 6t^2$

f) $4a - a^2 + 3a - 4 + 2a^2$

12. Simplify each polynomial.

a) $2m + 4 - 3m - 8$

b) $4 - 5x + 6x - 2$

c) $3g - 6 - 2g + 9$

d) $-5 + 1 + h - 4h$

e) $-6n - 5n - 4 - 7$

f) $3s - 4s - 5 - 6$

13. Simplify each polynomial.

a) $6 - 3x + x^2 + 9 - x$

b) $5m - 2m^2 - m^2 + 5m$

c) $5x - x^2 + 3x + x^2 - 7$

d) $3p^2 - 2p + 4 + p^2 + 3$

e) $a^2 - 2a - 4 + 2a - a^2 + 4$

f) $-6x^2 + 17x - 4 - 3x^2 + 8 - 12x$

14. Simplify each polynomial.

a) $3x^2 + 5y - 2x^2 - 1 - y$

b) $pq - 1 - p^2 + 5p - 5pq - 2p$

c) $5x^2 + 3xy - 2y - x^2 - 7x + 4xy$

d) $3r^2 - rs + 5s + r^2 - 2rs - 4s$

e) $4gh + 7 - 2g^2 - 3gh - 11 + 6g$

f) $-5s + st - 4s^2 - 12st + 10s - 2s^2$

15. Identify the equivalent polynomials. Justify your answers.

a) $1 + 5x$

b) $6 - 2x + x^2 - 1 - x + x^2$

c) $4x^2 - 7x + 1 - 7x^2 + 2x + 3$

d) $4 - 5x - 3x^2$

e) $2x^2 - 3x + 5$

f) $3x + 2x^2 + 1 - 2x^2 + 2x$

16. Write 3 different polynomials that simplify to $-2a^2 + 4a - 8$.

17. Write a polynomial with degree 2 and 5 terms, which has only 2 terms when it is simplified.

18. **Assessment Focus**

a) A student is not sure whether $x + x$ simplifies to $2x$ or x^2.
Explain how the student can use algebra tiles to determine the correct answer.
What is the correct answer?

b) Simplify each polynomial. How do you know that your answers are correct?

i) $-2 + 4r - 2r + 3$

ii) $2t^2 - 3t + 4t^2 - 6t$

iii) $3c^2 + 4c + 2 + c^2 + 2c + 1$

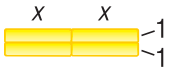
iv) $15x^2 - 12xy + 5y + 10xy - 8y - 9x^2$

c) Create a polynomial that cannot be simplified. Explain why it cannot be simplified.

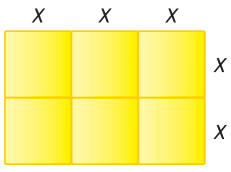
19. Write a polynomial to represent the perimeter of each rectangle.

a)

b)



c)



d)

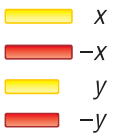
20. Each polynomial below represents the perimeter of a rectangle. Use algebra tiles to make the rectangle. Sketch the tiles. How many different rectangles can you make each time?

a) $6c + 4$ **b)** $4d$ **c)** $8 + 2m$

d) $12r$ **e)** $6s$ **f)** $4a + 10$

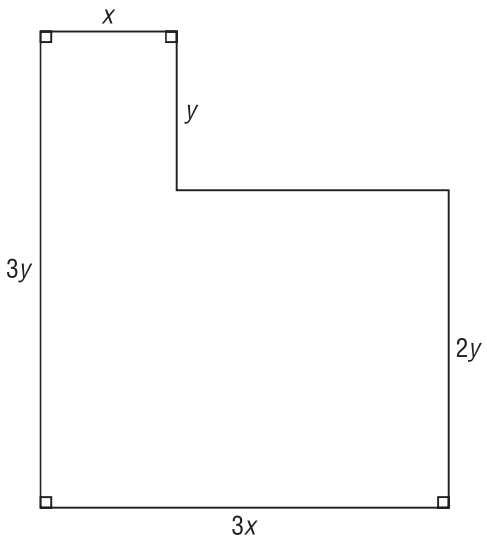
Take It Further

21. Many algebra tile kits contain x-tiles and y-tiles.



What do you think an xy-tile looks like? Sketch your idea and justify your picture.

22. Write a polynomial for the perimeter of this shape. Simplify the polynomial.



Reflect

Explain how like terms can be used to simplify a polynomial. Use diagrams and examples in your explanation.

Math Link

Your World

On a forward somersault dive, a diver's height above the water, in metres, in terms of the time t seconds after the diver leaves the board may be modelled by the polynomial $-4.9t^2 + 6t + 3$.