

5.3

Adding Polynomials

FOCUS

- Use different strategies to add polynomials.



Investigate

2

You will need algebra tiles and a paper bag.
Conduct the activity 3 times.

Put both colours of algebra tiles in a paper bag.
Each person removes a handful of algebra tiles and
writes the simplified polynomial that the tiles model.
Add the two polynomials.
Record your work as an addition sentence.



Reflect & Share

Compare your strategies for adding two polynomials with those of another pair of students.
If you used different strategies, explain your strategies.
If you used the same strategies, find a pair of students who used a different strategy.
Which terms can be combined when you add polynomials?
Why can these terms be combined?

Connect

To add polynomials, we combine the algebra tiles that represent each polynomial and record the process symbolically. This develops a strategy to add polynomials without algebra tiles.

When we write the sum of two polynomials, we write each polynomial in brackets.

To determine the sum of $3x^2 + 2x + 4$ and $-5x^2 + 3x - 5$, we write:

$$(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$$

Tile Model

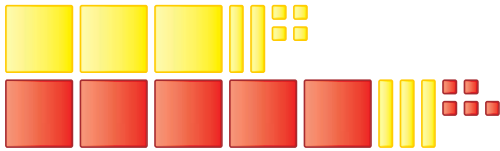
Display: $3x^2 + 2x + 4$



Display: $-5x^2 + 3x - 5$



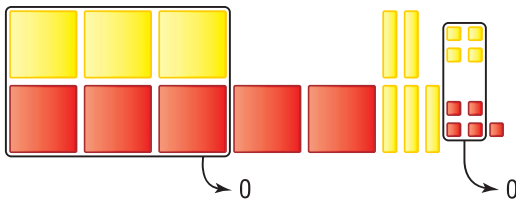
Combine the displays.



Group like tiles.



Remove zero pairs.



The remaining tiles represent

$$-2x^2 + 5x - 1.$$

Symbolic Record

The sum is:

$$(3x^2 + 2x + 4) + (-5x^2 + 3x - 5)$$

This is written as:

$$3x^2 + 2x + 4 - 5x^2 + 3x - 5$$

Group like terms:

$$3x^2 - 5x^2 + 2x + 3x + 4 - 5$$

Combine like terms:

$$-2x^2 + 5x - 1$$

Example 1 Adding Polynomials Symbolically

Add: $(7s + 14) + (-6s^2 + s - 6)$

Solutions

Add the polynomials by adding the coefficients of like terms. In the second polynomial, the term s has coefficient 1, so write s as $1s$.

Method 1

Add horizontally.

$$\begin{aligned}(7s + 14) + (-6s^2 + 1s - 6) & \quad \text{Remove the brackets.} \\= 7s + 14 - 6s^2 + 1s - 6 & \quad \text{Group like terms.} \\= -6s^2 + 7s + 1s + 14 - 6 & \quad \text{Combine like terms by adding their coefficients.} \\= -6s^2 + 8s + 8\end{aligned}$$

Method 2

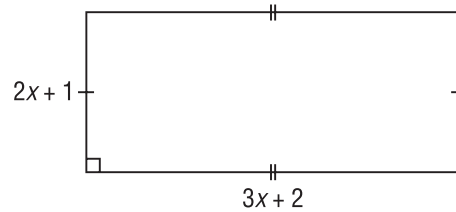
Add vertically. Align like terms, then add their coefficients.

$$\begin{array}{r}7s + 14 \\+ -6s^2 + 1s - 6 \\ \hline-6s^2 + 8s + 8\end{array}$$

So, $(7s + 14) + (-6s^2 + 1s - 6) = -6s^2 + 8s + 8$

Example 2 Determining a Polynomial for the Perimeter of a Rectangle

- a) Write a polynomial for the perimeter of this rectangle.
Simplify the polynomial.



- b) Substitute to check the answer.

A Solution

- a) The perimeter is the sum of the measures of the four sides.

$$\begin{array}{r}2x + 1 \\+ 2x + 1 \\+ 3x + 2 \\+ 3x + 2 \\ \hline10x + 6\end{array}$$

The perimeter is $10x + 6$.

b) Choose a value for x , such as $x = 1$.

Write the addition sentence:

$$2x + 1 + 2x + 1 + 3x + 2 + 3x + 2 = 10x + 6$$

Substitute $x = 1$.

Left side:

$$\begin{aligned} & 2x + 1 + 2x + 1 + 3x + 2 + 3x + 2 \\ &= 2(1) + 1 + 2(1) + 1 + 3(1) + 2 + 3(1) + 2 \\ &= 2 + 1 + 2 + 1 + 3 + 2 + 3 + 2 \\ &= 16 \end{aligned}$$

Right side:

$$\begin{aligned} 10x + 6 &= 10(1) + 6 \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

Since the left side equals the right side, the polynomial for the perimeter is correct.

Example 3 Adding Polynomials in Two Variables

Add: $(2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2)$

A Solution

$$\begin{aligned} & (2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2) \\ &= 2a^2 + a - 3b - 7ab + 3b^2 - 4b^2 + 3ab + 6b - 5a + 5a^2 \\ &= 2a^2 + 5a^2 + a - 5a - 3b + 6b - 7ab + 3ab + 3b^2 - 4b^2 \\ &= 7a^2 - 4a + 3b - 4ab - b^2 \end{aligned}$$

Remove brackets.
Group like terms.
Combine like terms.

Discuss the ideas

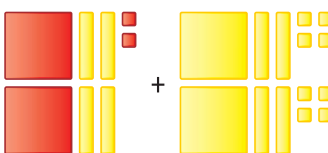
1. How can you use what you know about adding integers to add polynomials?
2. How is adding polynomials like simplifying a polynomial?

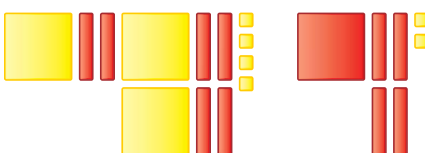
Practice

Check

3. Write the polynomial sum modelled by each set of tiles.

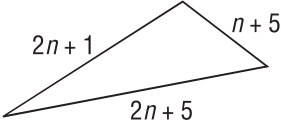
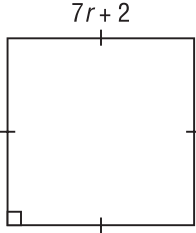
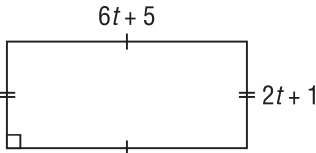
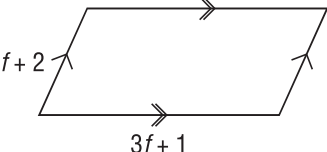
a) 

b) 

c) 

4. Explain how to use algebra tiles to determine $(3x^2 + 2) + (x^2 - 1)$. What is the sum?
5. Use algebra tiles to model each sum of binomials. Record your answer symbolically.
- $(5g + 3) + (2g + 4)$
 - $(3 - 2j) + (-4 + 2j)$
 - $(p + 1) + (5p - 6)$
 - $(7 + 4m) + (-5m + 4)$
6. Add these polynomials. Visualize algebra tiles if it helps.
- $$\begin{array}{r} 2x + 4 \\ + 3x - 5 \\ \hline \end{array}$$
 - $$\begin{array}{r} 3x^2 + 5x \\ + -2x^2 - 8x \\ \hline \end{array}$$
 - $$\begin{array}{r} 3x^2 + 5x + 7 \\ + -8x^2 - 3x + 5 \\ \hline \end{array}$$
7. Do you prefer to add vertically or horizontally? Give reasons for your choice.

Apply

8. Use a personal strategy to add.
- $(6x + 3) + (3x + 4)$
 - $(5b - 4) + (2b + 9)$
 - $(6 - 3y) + (-3 - 2y)$
 - $(-n + 7) + (3n - 2)$
 - $(-4s - 5) + (6 - 3s)$
 - $(1 - 7h) + (-7h - 1)$
 - $(8m + 4) + (-9 + 3m)$
 - $(-8m - 4) + (9 - 3m)$
9. Add. Which strategy did you use each time?
- $(4m^2 + 4m - 5) + (2m^2 - 2m + 1)$
 - $(3k^2 - 3k + 2) + (-3k^2 - 3k + 2)$
 - $(-7p - 3) + (p^2 + 5)$
 - $(9 - 3t) + (9t + 3t^2 - 6t)$
 - $(3x^2 - 2x + 3) + (2x^2 + 4)$
 - $(3x^2 - 7x + 5) + (6x - 6x^2 + 8)$
 - $(6 - 7x + x^2) + (6x - 6x^2 + 10)$
 - $(1 - 3r + r^2) + (4r + 5 - 3r^2)$
10. a) For each shape below, write the perimeter:
- as a sum of polynomials
 - in simplest form
- 
 - 
 - 
 - 
- b) Use substitution to check each answer in part a.
11. Sketch 2 different shapes whose perimeter could be represented by each polynomial.
- $8 + 6r$
 - $3s + 9$
 - $4 + 12t$
 - $20u$
 - $7 + 5v$
 - $4y + 6$
 - $9 + 9c$
 - $15m$

12. A student added $(4x^2 - 7x + 3)$ and $(-x^2 - 5x + 9)$ as follows.

Handwritten work showing the addition of two polynomials:

$$\begin{aligned} &(4x^2 - 7x + 3) + (-x^2 - 5x + 9) \\ &= 4x^2 - 7x + 3 - x^2 - 5x + 9 \\ &= 4x^2 - x^2 - 7x - 5x + 3 + 9 \\ &= 3x^2 - 2x + 1 \end{aligned}$$

Is the student's work correct?
If not, explain where the student made any errors and write the correct answer.

13. Assessment Focus

These tiles represent the sum of two polynomials.



- a) What might the two polynomials be?
Explain how you found out.
- b) How many different pairs of polynomials can you find? List all the pairs you found.
14. The sum of two polynomials is $12m^2 + 2m + 4$.
One polynomial is $4m^2 - 6m + 8$.
What is the other polynomial?
Explain how you found your answer.

15. Create a polynomial that is added to $3x^2 + 7x + 2$ to get each sum.

- a) $5x^2 + 10x + 1$ b) $2x^2 + 5x + 8$
c) $4x^2 + 3x$ d) $-x^2 + x - 1$
e) $2x + 3$ f) 4

16. a) What polynomial must be added to $5x^2 + 3x - 1$ to obtain a sum of 0? Justify your answer.
b) How are the coefficients of the two polynomials related?
Will this relationship be true for all polynomials with a sum of 0? Explain.

17. Add.

- a) $(3x^2 - 2y^2 + xy) + (-2xy - 2y^2 - 3x^2)$
b) $(-5q^2 + 3p - 2q + p^2) + (4p + q + pq)$
c) $(3mn + m^2 - 3n^2 + 5m) + (7n^2 - 8n + 10)$
d) $(3 - 8f + 5g - f^2) + (2g^2 - 3f + 4g - 5)$

Take It Further

18. a) The polynomials $4x - 3y$ and $2x + y$ represent the lengths of two sides of a triangle. The perimeter of the triangle is $9x + 2$. Determine the length of the third side.
b) Use substitution to check your solution in part a.
19. The polynomial $5y + 3x + 7$ represents the perimeter of an isosceles triangle. Write three polynomials that could represent the side lengths of the triangle. Find as many answers as you can.

Reflect

What strategies can you use for adding polynomials?
Which strategy do you prefer?
How can you check that your answers are correct?
Include examples in your explanation.