5.4 Subtracting Polynomials

What strategies do you know to subtract two integers, such as $-2 - 3$?
How could these strategies help you subtract two polynomials?

FOCUS

- Use different strategies to subtract polynomials.

Investigate

2

Use algebra tiles.

➤ Write two like monomials.
Subtract the monomials.
Write the subtraction sentence.
Subtract the monomials in the reverse order.
Write the new subtraction sentence.
Sketch the tiles you used.

➤ Repeat the process above for two binomials, then for two trinomials.

➤ Subtract. Use a strategy of your choice.
$(5x) - (3x)$
$(2x^2 + 3x) - (4x^2 - 6x)$
$(3x^2 - 6x + 4) - (x^2 + 3x - 2)$
Use a different strategy to verify your answer.

Reflect & Share

Compare your answers and strategies with those of a pair of students who used a different strategy.
Explain your strategies to each other.
Work together to write an addition sentence that corresponds to each subtraction sentence.

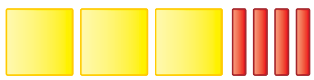
Connect

Here are two strategies to subtract polynomials.

➤ Using algebra tiles

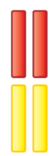
To subtract: $(3x^2 - 4x) - (2x^2 - 6x)$

Use algebra tiles to model $3x^2 - 4x$.

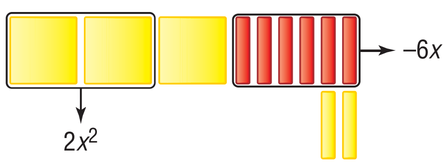


To subtract $2x^2 - 6x$, we need to:

- Take away two x^2-tiles from three x^2-tiles.
- Take away six $-x$-tiles from four $-x$-tiles.
  To do this, we need 2 more $-x$-tiles.
  So, we add 2 zero pairs of x-tiles.



Now we can take away the tiles for $2x^2 - 6x$.



The remaining tiles represent $x^2 + 2x$.

So, $(3x^2 - 4x) - (2x^2 - 6x) = x^2 + 2x$

➤ Using the properties of integers

We know that -6 is the opposite of 6.

Subtracting -6 from an integer is the same as adding 6 to that integer.

The same process is true for like terms.

To subtract: $(3x^2 - 4x) - (2x^2 - 6x)$

$(3x^2 - 4x) - (2x^2 - 6x) = 3x^2 - 4x - (2x^2) - (-6x)$	Subtract each term.
$= 3x^2 - 4x - 2x^2 - (-6x)$	Add the opposite term.
$= 3x^2 - 4x - 2x^2 + 6x$	Collect like terms.
$= 3x^2 - 2x^2 - 4x + 6x$	Combine like terms.
$= x^2 + 2x$	

Example 1 Subtracting Two Trinomials

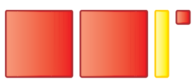
Subtract: $(-2a^2 + a - 1) - (a^2 - 3a + 2)$

Solutions

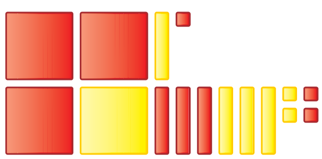
$(-2a^2 + a - 1) - (a^2 - 3a + 2)$

Method 1

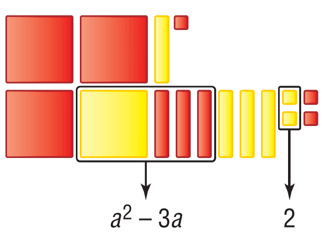
Use algebra tiles.
Display: $-2a^2 + a - 1$



To subtract a^2, add a zero pair of a^2-tiles.
To subtract $-3a$, add 3 zero pairs of a-tiles.
To subtract 2, add 2 zero pairs of 1-tiles.



Now remove tiles for $a^2 - 3a + 2$.



The remaining tiles represent $-3a^2 + 4a - 3$.

Method 2

Use the properties of integers.

$$\begin{aligned}
&(-2a^2 + a - 1) - (a^2 - 3a + 2)\\
&= -2a^2 + a - 1 - (a^2) - (-3a) - (+2)\\
&= -2a^2 + a - 1 - a^2 + 3a - 2\\
&= -2a^2 - a^2 + a + 3a - 1 - 2\\
&= -3a^2 + 4a - 3
\end{aligned}$$

To check the difference when two numbers are subtracted,
we add the difference to the number that was subtracted;
for example, to check that $23 - 5 = 18$ is correct, we add: $5 + 18 = 23$

We can use the same process to check the difference of two polynomials.

Example 2 Subtracting Trinomials in Two Variables

Subtract: $(5x^2 - 3xy + 2y^2) - (8x^2 - 7xy - 4y^2)$
Check the answer.

A Solution

$$\begin{aligned}(5x^2 - 3xy + 2y^2) - (8x^2 - 7xy - 4y^2) &= 5x^2 - 3xy + 2y^2 - (8x^2) - (-7xy) - (-4y^2)\\ &= 5x^2 - 3xy + 2y^2 - 8x^2 + 7xy + 4y^2\\ &= 5x^2 - 8x^2 - 3xy + 7xy + 2y^2 + 4y^2\\ &= -3x^2 + 4xy + 6y^2\end{aligned}$$

To check, add the difference to the second polynomial:

$$\begin{aligned}(-3x^2 + 4xy + 6y^2) + (8x^2 - 7xy - 4y^2) &= -3x^2 + 4xy + 6y^2 + 8x^2 - 7xy - 4y^2\\ &= -3x^2 + 8x^2 + 4xy - 7xy + 6y^2 - 4y^2\\ &= 5x^2 - 3xy + 2y^2\end{aligned}$$

The sum is equal to the first polynomial.
So, the difference is correct.

Discuss the ideas

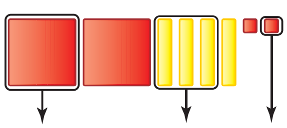
1. How is subtracting polynomials like subtracting integers?
2. How is subtracting polynomials like adding polynomials? How is it different?
3. When might using algebra tiles not be the best method to subtract polynomials?

Practice

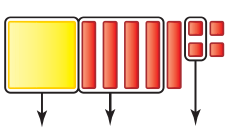
Check

4. Write the subtraction sentence that these algebra tiles represent.

a)



b)



5. Use algebra tiles to subtract.
Sketch the tiles you used.

a) $(5r) - (3r)$
b) $(5r) - (-3r)$
c) $(-5r) - (3r)$
d) $(-5r) - (-3r)$
e) $(3r) - (5r)$
f) $(-3r) - (5r)$
g) $(3r) - (-5r)$
h) $(-3r) - (-5r)$

Apply

6. Use algebra tiles to model each difference of binomials. Record your answer symbolically.

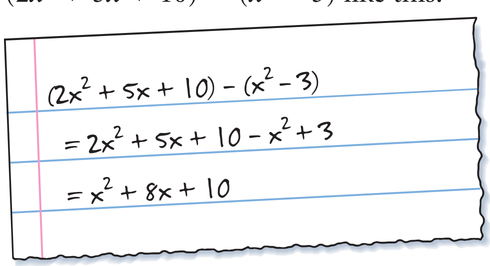
a) $(5x + 3) - (3x + 2)$
b) $(5x + 3) - (3x - 2)$
c) $(5x + 3) - (-3x + 2)$
d) $(5x + 3) - (-3x - 2)$

7. Use algebra tiles to model each difference of trinomials. Record your answer symbolically.
   a) $(3s^2 + 2s + 4) - (2s^2 + s + 1)$
   b) $(3s^2 - 2s + 4) - (2s^2 - s + 1)$
   c) $(3s^2 - 2s - 4) - (-2s^2 + s - 1)$
   d) $(-3s^2 + 2s - 4) - (2s^2 - s - 1)$

8. Use a personal strategy to subtract. Check your answers by adding.
   a) $(3x + 7) - (-2x - 2)$
   b) $(b^2 + 4b) - (-3b^2 + 7b)$
   c) $(-3x + 5) - (4x + 3)$
   d) $(4 - 5p) - (-7p + 3)$
   e) $(6x^2 + 7x + 9) - (4x^2 + 3x + 1)$
   f) $(12m^2 - 4m + 7) - (8m^2 + 3m - 3)$
   g) $(-4x^2 - 3x - 11) - (x^2 - 4x - 15)$
   h) $(1 - 3r + r^2) - (4r + 5 - 3r^2)$

9. The polynomial $4n + 2500$ represents the cost, in dollars, to produce n copies of a magazine in colour. The polynomial $2n + 2100$ represents the cost, in dollars, to produce n copies of the magazine in black-and-white.
   a) Write a polynomial for the difference in the costs of the two types of magazines.
   b) Suppose the company wants to print 3000 magazines. How much more does it cost to produce the magazine in colour instead of black-and-white?

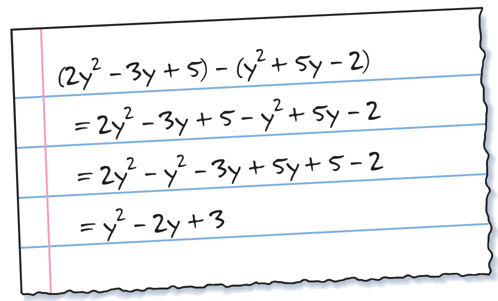
10. A student subtracted $(2x^2 + 5x + 10) - (x^2 - 3)$ like this:



   a) Use substitution to show that the answer is incorrect.
   b) Identify the errors and correct them.

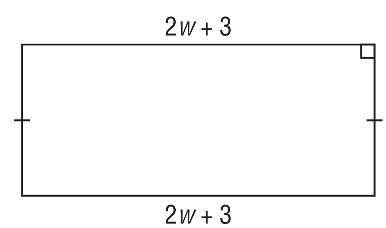
11. **Assessment Focus** Create a polynomial subtraction question. Answer your question. Check your answer. Show your work.

12. A student subtracted like this:

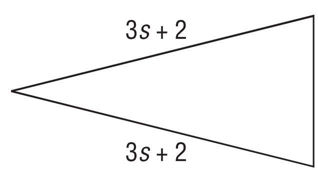


   a) Explain why the solution is incorrect.
   b) What is the correct answer? Show your work.
   c) How could you check that your answer is correct?
   d) What could the student do to avoid making the same mistakes in the future?

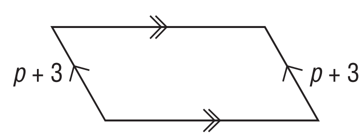
13. The perimeter of each polygon is given. Determine each unknown length.
   a) $6w + 14$



   b) $7s + 7$



   c) $10p + 8$



14. a) Write two polynomials, then subtract them.

b) Subtract the polynomials in part a in the reverse order.

c) How do the answers in parts a and b compare? Why are the answers related this way?

15. Subtract.

a) $(r^2 - 3rs + 5s^2) - (-2r^2 - 3rs - 5s^2)$

b) $(-3m^2 + 4mn - n^2) - (5m^2 + 7mn + 2n^2)$

c) $(5cd + 8c^2 - 7d^2) - (3d^2 + 6cd - 4c^2)$

d) $(9e + 9f - 3e^2 + 4f^2) - (-f^2 - 2e^2 + 3f - 6e)$

e) $(4jk - 7j - 2k + k^2) - (2j^2 + 3j - jk)$

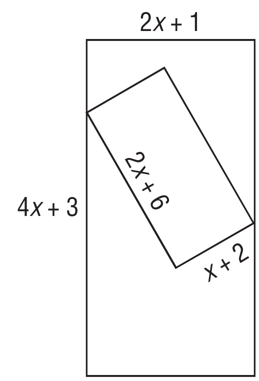
16. The difference of two polynomials is $3x^2 + 4x - 7$.
One polynomial is $-8x^2 + 5x - 4$.

a) What is the other polynomial?

b) Why are there two possible answers to part a?

Take It Further

17. The diagram shows one rectangle inside another rectangle. What is the difference in the perimeters of the rectangles?



18. One polynomial is subtracted from another. The difference is $-4x^2 + 2x - 5$.
Write two polynomials that have this difference. How many different pairs of polynomials can you find? Explain.

Reflect

What strategy or strategies do you use to subtract polynomials?
Why do you prefer this strategy or strategies?

Math Link

Your World

On a suspension bridge, the roadway is hung from huge cables passing through the tops of high towers. Here is a photograph of the Lions Gate Bridge in Vancouver. The position of any point on the cable can be described by its horizontal and vertical distance from the centre of the bridge. The vertical distance in metres is modelled by the polynomial $0.0006x^2$, where x is the horizontal distance in metres.

Mid-Unit Review

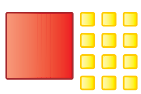
5.1

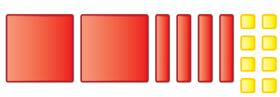
1. In each polynomial, identify:
the variable, number of terms, coefficients, constant term, and degree.

a) $3m - 5$
b) $4r$
c) $x^2 + 4x + 1$

2. Create a polynomial that meets these conditions:
trinomial in variable m, degree 2, constant term is -5

3. Which polynomial is represented by each set of algebra tiles? Is the polynomial a monomial, binomial, or trinomial? How do you know?

a) 

b) 

c) 

4. Use algebra tiles to represent each polynomial. Sketch the tiles you used.
a) $4n - 2$
b) $-t^2 + 4t$
c) $2d^2 + 3d + 2$

5.2

5. For each pair of monomials, which are like terms? Explain how you know.
a) $2x, -5x$
b) $3, 4g$
c) $10, 2$
d) $2q^2, -7q^2$
e) $8x^2, 3x$
f) $-5x, -5x^2$

6. Simplify $3x^2 - 7 + 3 - 5x^2 - 3x + 5$.
Explain how you did this.

7. Renata simplified a polynomial and got $4x^2 + 2x - 7$. Her friend simplified the same polynomial and got $-7 + 4x^2 + 2x$. Renata thinks her friend's answer is wrong. Do you agree? Explain.

8. Cooper thinks that $5x - 2$ simplifies to $3x$. Is he correct? Explain.
Use algebra tiles to support your explanation.

9. Identify the equivalent polynomials.
Justify your answers.
a) $1 + 3x - x^2$
b) $1 + 3x^2 - x^2 + 2x - 2x^2 + x - 2$
c) $x^2 - 3x - 1$
d) $6 + 6x - 6x^2 - 4x - 5 + 2x^2 + x^2 - 4$
e) $3x - 1$
f) $-3x^2 + 2x - 3$
g) $6x^2 - 6x - 6 + x - 5x^2 - 1 + 2x + 4$
h) $3x - x^2 + 1$

5.3
5.4

10. Use algebra tiles to add or subtract.
Sketch the tiles you used.
a) $(4f^2 - 4f) + (-2f^2)$
b) $(3r^2 + 2r + 5) + (-7r^2 + r - 3)$
c) $(-2v + 5) - (-9v + 3)$
d) $(-2g^2 - 12) - (-6g^2 + 4g - 1)$

11. Add or subtract. Use a strategy of your choice.
a) $(3w^2 + 17w) + (12w^2 - 3w)$
b) $(5m^2 - 3) + (m^2 + 3)$
c) $(-3h - 12) - (-9h - 6)$
d) $(6a^2 + 2a - 2) + (-7a^2 + 4a + 11)$
e) $(3y^2 + 9y + 7) - (2y^2 - 4y + 13)$
f) $(-14 + 3p^2 + 2p) - (-5p + 10 - 7p^2)$

12. a) Which polynomial must be added to $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?
b) Which polynomial must be subtracted from $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?
Justify your answers.

Start Where You Are

How Can I Summarize What I Have Learned?

Suppose I want to summarize what I know about polynomials.

➤ What tools could I use to do this?

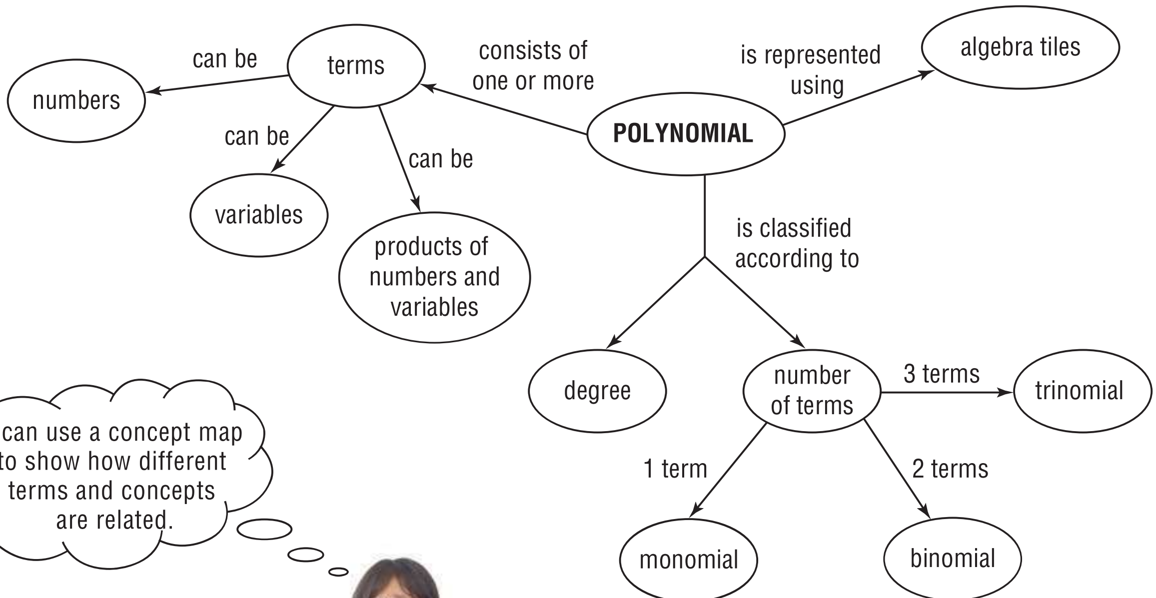
- a Frayer model
- a table
- a concept map

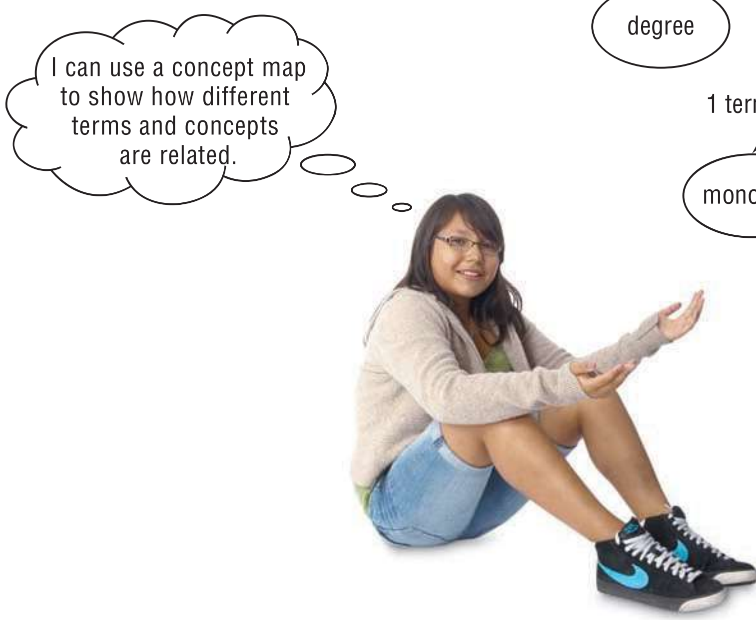
I can use a Frayer model to explain the meaning of a term or concept.

Definition Like terms have the same variable raised to the same exponent.	**Facts/Characteristics** Like terms are represented by algebra tiles with the same size and shape. I can combine like terms by adding their coefficients.
Like terms	
Examples $-3x$ and $4x$ $5b^2$ and $2b^2$	**Non-examples** $-3c$ and 4 $5n^2$ and $2n$

I can use a table to show how terms and concepts are alike and different.

Polynomial	Number of Terms	Name by Number of Terms	Degree
12	1	monomial	0
$8a$	1	monomial	1
$-4b^2 + 9$	2	binomial	2
$2c - 7$	2	binomial	1
$3d^2 - 4d + 6$	3	trinomial	2





Check

Use the tools *you* find most helpful to summarize the important ideas and concepts you have learned about polynomials.

1. Choose another term or concept. Make a Frayer model to show what you know about that term or concept.

2. What other types of polynomials could you include in the table on page 238?

3. a) What could you add to the concept map above?
b) Think of another way to draw a concept map about polynomials.

Add to your Frayer model, table, or concept map as you work through this unit.

GAME

Investigating Polynomials that Generate Prime Numbers

A prime number is any whole number, greater than 1, that is divisible by only itself and 1.

In 1772, Leonhard Euler, a Swiss mathematician, determined that the polynomial $n^2 - n + 41$ generates prime numbers for different values of n.

Use a calculator to check that this is true:

- ➤ Choose a value of n between 1 and 10.
  Substitute this number for n in the polynomial.
  Is the number you get a prime number?
  How do you know?
- ➤ Repeat the process for other values of n between 1 and 10.
- ➤ Choose a value of n between 10 and 40.
  Substitute this number for n in the polynomial.
  Is the number you get a prime number?
  How do you know?
- ➤ Repeat the process for other values of n between 10 and 40.
- ➤ Substitute $n = 41$. Is the number you get a prime number?
  How can you tell?
- ➤ List the values of n and the resulting primes in a table.

In 1879, E. B. Escott, an American mathematician, determined the polynomial $n^2 - 79n + 1601$ for generating prime numbers.

Test this polynomial:

- ➤ Substitute different values of n, and check that the numbers you get are prime. List the values of n and the resulting primes in a table. What patterns do you see?
- ➤ Substitute $n = 80$. Did you get a prime number? Explain.
- ➤ Determine other values of n for which Escott's polynomial does *not* generate prime numbers.

Currently, there is no known polynomial that generates only prime numbers. And, there is no known polynomial that generates all the prime numbers.

- ➤ Determine a value of n for which each of these polynomials does *not* generate a prime number:
  - $n^2 - n + 41$, other than $n = 41$
  - $n^2 - n + 17$
  - $n^2 + n - 1$