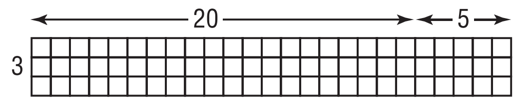
5.5 Multiplying and Dividing a Polynomial by a Constant

FOCUS

- Use different strategies to multiply and divide a polynomial by a constant.

How does this diagram model the product 3×25?



What property is illustrated by this diagram?
How could you use the diagram above to model division?

Investigate

2

Use any strategy or materials you wish.

➤ Determine each product. Write a multiplication sentence.
- $2(3x)$
- $3(2x + 1)$
- $2(2x^2 + x + 4)$
- $-2(3x)$
- $-3(2x + 1)$
- $-2(2x^2 + x + 4)$

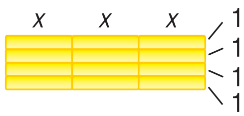
➤ Determine each quotient. Write a division statement.
- $9x \div 3$
- $(8x + 12) \div 4$
- $(5x^2 + 10x + 20) \div 5$
- $9x \div (-3)$
- $(8x + 12) \div (-4)$
- $(5x^2 + 10x + 20) \div (-5)$

Reflect & Share

Compare your answers and strategies with those of another pair of students.
If your answers are different, find out why.
Look at your multiplication and division sentences.
What relationships do you see among the original terms and the answers?
How could you use these relationships to multiply and divide without using algebra tiles?

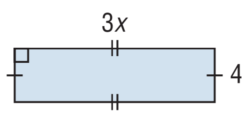
Connect

➤ The expression $4(3x)$ is a product statement.
It represents the product of the constant, 4, and the monomial, $3x$.
We can model the product as 4 rows of three x-tiles.



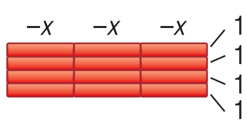
So, $4(3x) = 3x + 3x + 3x + 3x$ This is repeated addition.
$= 12x$

We can also model $4(3x)$ as the area of a rectangle with dimensions 4 and $3x$.



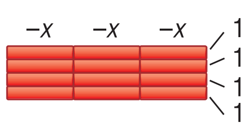
So, $4(3x) = 4(3)(x)$
$= 12x$

➤ $4(-3x)$ is the product of 4 and the monomial $-3x$.
We can model the product as 4 rows of three $-x$-tiles.



So, $4(-3x) = -3x - 3x - 3x - 3x$
$= -12x$

➤ $-4(3x)$ is the opposite of $4(3x)$.
We can model this by flipping the tiles we used to model $4(3x)$.



So, $-4(3x) = -(12x)$
$= -12x$

We can use the same strategy with algebra tiles to multiply a binomial or a trinomial by a constant. To determine the product symbolically, we use the *distributive property*.

Example 1 Multiplying a Binomial and a Trinomial by a Constant

Determine each product.

a) $3(-2m + 4)$ **b)** $-2(-n^2 + 2n - 1)$

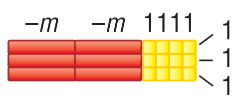
Solutions

Method 1

Use algebra tiles.

a) $3(-2m + 4)$

Display 3 rows of two $-m$-tiles and four 1-tiles.

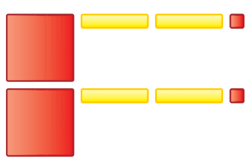


There are six $-m$-tiles and twelve 1-tiles.

So, $3(-2m + 4) = -6m + 12$

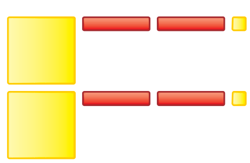
b) $-2(-n^2 + 2n - 1)$

Display 2 rows of one $-n^2$-tile, two n-tiles, and one –1-tile.



This shows $2(-n^2 + 2n - 1)$.

Flip all the tiles.



There are two n^2-tiles, four $-n$-tiles, and two 1-tiles.

So, $-2(-n^2 + 2n - 1) = 2n^2 - 4n + 2$

Method 2

Use the distributive property.

Multiply each term in the brackets by the term outside the brackets.

a) $3(-2m + 4) = 3(-2m) + 3(4)$

$= -6m + 12$

b) $-2(-n^2 + 2n - 1)$

$= (-2)(-1n^2) + (-2)(2n) + (-2)(-1)$

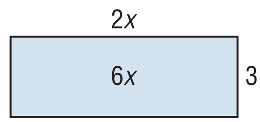
$= 2n^2 + (-4n) + 2$

$= 2n^2 - 4n + 2$

Multiplication and division are inverse operations. To divide a polynomial by a constant, we reverse the process of multiplication.

➤ The expression $6x \div 3$ is a division statement.
It represents the quotient of the monomial, $6x$, and the constant 3.
To model $6x \div 3$,
we arrange six x-tiles in 3 rows.
Each row contains two x-tiles.
So, $6x \div 3 = 2x$

We can also model $6x \div 3$ as
one dimension of a rectangle with
an area of $6x$ and the other dimension 3.
Then, $6x \div 3 = \frac{6x}{3}$
$= 2x$

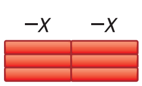


We can use what we know about division as a fraction and integer division to determine the quotient.

$$\begin{aligned}\frac{6x}{3} &= \frac{6}{3} \times x \\ &= 2 \times x \\ &= 2x\end{aligned}$$

➤ $(-6x) \div 3$ is the quotient of the monomial, $-6x$, and the constant 3.

Using a model:
We arrange six $-x$-tiles in 3 rows.



Each row contains two $-x$-tiles.
So, $(-6x) \div 3 = -2x$

Using fractions and integers:
$(-6x) \div 3 = \frac{-6x}{3}$
Simplify the fraction.

$$\begin{aligned}(-6x) \div 3 &= \frac{-6}{3} \times x \\ &= -2 \times x \\ &= -2x\end{aligned}$$

➤ $6x \div (-3)$ is the quotient of the monomial, $6x$, and the constant -3.
Using fractions and integers:
$6x \div (-3) = \frac{6x}{-3}$
Simplify the fraction.

$$\begin{aligned}6x \div (-3) &= \frac{6}{-3} \times x \\ &= -2 \times x \\ &= -2x\end{aligned}$$

Example 2 Dividing a Binomial and a Trinomial by a Constant

Determine each quotient.

a) $\frac{4s^2 - 8}{4}$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

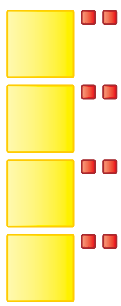
Solutions

Method 1

a) $\frac{4s^2 - 8}{4}$

Use algebra tiles.

Arrange four s^2-tiles and eight -1-tiles in 4 equal rows.



In each row, there is one s^2-tile and two -1-tiles.

So, $\frac{4s^2 - 8}{4} = s^2 - 2$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

Think multiplication.

What do we multiply -3 by to get $-3m^2 + 15mn - 21n^2$?

$(-3) \times ? = -3m^2 + 15mn - 21n^2$

Since $(-3) \times 1 = -3$,

then $(-3) \times (1m^2) = -3m^2$

Since $(-3) \times (-5) = 15$,

then $(-3) \times (-5mn) = +15mn$

Since $(-3) \times 7 = -21$,

then $(-3) \times (+7n^2) = -21n^2$

So, $\frac{-3m^2 + 15mn - 21n^2}{-3} = m^2 - 5mn + 7n^2$

Method 2

a) $\frac{4s^2 - 8}{4}$

Write the quotient expression as the sum of 2 fractions.

$\frac{4s^2 - 8}{4} = \frac{4s^2}{4} + \frac{-8}{4}$

Simplify each fraction.

$= \frac{4}{4} \times s^2 + (-2)$

$= 1 \times s^2 - 2$

$= s^2 - 2$

b) $\frac{-3m^2 + 15mn - 21n^2}{-3}$

Write the quotient expression as the sum of 3 fractions.

$\frac{-3m^2 + 15mn - 21n^2}{-3}$

$= \frac{-3m^2}{-3} + \frac{15mn}{-3} + \frac{-21n^2}{-3}$

Simplify each fraction.

$= m^2 + (-5mn) + (7n^2)$

$= m^2 - 5mn + 7n^2$

Discuss the ideas

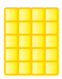
1. How could you use multiplication to verify the quotient in a division question?

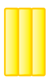
2. Why can we not use algebra tiles to divide when the divisor is negative?

Practice

Check

3. Write the multiplication sentence modelled by each set of algebra tiles.

   a) 

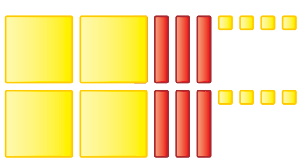
   b) 

   c) 

   d) 

4. For each set of algebra tiles in question 3, write a division sentence.

5. a) Which of these products is modelled by the algebra tiles below?
      i) $2(-2n^2 + 3n + 4)$
      ii) $2(2n^2 - 3n + 4)$
      iii) $-2(2n^2 - 3n + 4)$

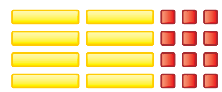
   b) In part a, two of the products were not modelled by the algebra tiles. Model each product. Sketch the tiles you used.

6. Which of these quotients is modelled by the algebra tiles below?

   a) $\frac{8t - 12}{-4}$

   b) $\frac{-8t - 12}{4}$

   c) $\frac{8t - 12}{4}$

Apply

7. a) Multiply.
      i) $3(5r)$
      ii) $-3(5r)$
      iii) $(5r)(3)$
      iv) $-5(3r)$
      v) $-5(-3r)$
      vi) $(-3r)(5)$

   b) In part a, explain why some answers are the same.

   c) For which products in part a could you have used algebra tiles? For each product, sketch the tiles you could use.

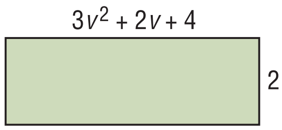
8. a) Divide.
      i) $\frac{12k}{4}$
      ii) $(-12k) \div 4$
      iii) $\frac{12k}{-4}$
      iv) $(-12k) \div (-4)$

   b) In part a, explain why some answers are the same.

   c) For which quotients in part a could you have used algebra tiles? For each quotient, sketch the tiles you could use.

9. Write the multiplication sentence modelled by each rectangle.

a)



b)

10. For each rectangle in question 9, write a division sentence.

11. Use algebra tiles to determine each product. Sketch the tiles you used. Record the product symbolically.

a) $7(3s + 1)$

b) $-2(-7h + 4)$

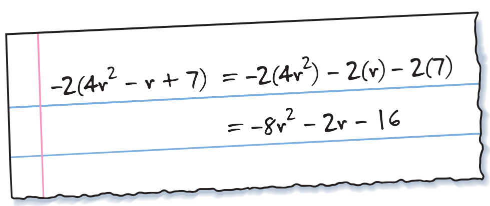
c) $2(-3p^2 - 2p + 1)$

d) $-6(2v^2 - v + 5)$

e) $(-w^2 + 3w - 5)(3)$

f) $(x^2 + x)(-5)$

12. Here is a student's solution for this question:



Identify the errors in the solution, then write the correct solution.

13. Use algebra tiles to determine each quotient. Sketch the tiles you used. Record the product symbolically.

a) $\frac{12p - 18}{6}$

b) $\frac{-6q^2 - 10}{2}$

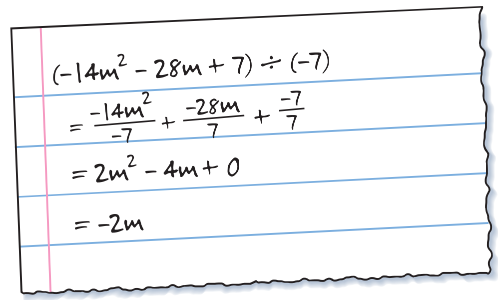
c) $\frac{5h^2 - 20h}{5}$

d) $\frac{4r^2 - 16r + 6}{2}$

e) $\frac{-8a^2 + 4a - 12}{4}$

f) $\frac{6x^2 + 3x + 9}{3}$

14. Here is a student's solution for this question:
Divide: $(-14m^2 - 28m + 7) \div (-7)$



Identify the errors in the solution, then write the correct solution.

15. Use any strategy to determine each product.

a) $-3(-4u^2 + 16u + 8)$

b) $12(2m^2 - 3m)$

c) $(5t^2 + 2t)(-4)$

d) $(-6s^2 - 5s - 7)(-5)$

e) $4(-7y^2 + 3y - 9)$

f) $10(8n^2 - n - 6)$

16. Use any strategy to determine each quotient.

a) $\frac{24d^2 - 12}{12}$

b) $\frac{8x + 4}{4}$

c) $\frac{-10 + 4m^2}{-2}$

d) $(25 - 5n) \div (-5)$

e) $(-14k^2 + 28k - 49) \div 7$

f) $\frac{30 - 36d^2 + 18d}{-6}$

g) $\frac{-26c^2 + 39c - 13}{-13}$

17. Which pairs of expressions are equivalent? Explain how you know.

a) $5j^2 + 4$ and $5(j + 4)$

b) $10x^2$ and $3x(x + 7)$

c) $15x - 10$ and $5(-2 + 3x)$

d) $-3(-4x - 1)$ and $12x^2 - 3x$

e) $-5(3x^2 - 7x + 2)$ and $-15x^2 + 12x - 10$

f) $2x(-3x - 7)$ and $-6x^2 - 14x$

18. **Assessment Focus**

a) Determine each product or quotient.

i) $(3p)(4)$ ii) $\frac{-21x}{3}$

iii) $(3m^2 - 7)(-4)$

iv) $\frac{-2f^2 + 14f - 8}{2}$

v) $(6y^2 - 36y) \div (-6)$

vi) $(-8n + 2 - 3n^2)(3)$

b) List the products and quotients in part a that can be modelled with algebra tiles. Justify your selection.

c) Sketch the tiles for one product and one quotient in part a.

19. a) Determine each product.

i) $2(2x + 1)$ ii) $2(1 - 2x)$
$3(2x + 1)$ $3(1 - 2x)$
$4(2x + 1)$ $4(1 - 2x)$
$5(2x + 1)$ $5(1 - 2x)$

b) Describe the patterns in part a.

c) Predict the next 3 products in each list in part a. How do you know the products are correct?

d) Suppose you extended the lists in part a upward. Predict the preceding 3 products in each list.

20. a) The perimeter of an equilateral triangle is represented by the polynomial $15a^2 + 21a + 6$.
Determine the polynomial that represents the length of one side.

b) Determine the length of one side when $a = 4$ cm.

21. Square A has side length $4s + 1$. Square B has a side length that is 3 times as great as the side length of square A.

a) What is the perimeter of each square? Justify your answers.

b) Write a polynomial, in simplest form, to represent the difference in the perimeters of squares A and B.

22. Determine each product.

a) $2(2x^2 - 3xy + 7y^2)$

b) $-4(pq + 3p^2 + 3q^2)$

c) $(-2gh + 6h^2 - 3g^2 - 9g)(3)$

d) $5(-r^2 + 8rs - 3s^2 - 5s + 4r)$

e) $-2(4t^2 - 3v^2 + 19tv - 6v - t)$

23. Determine each quotient.

a) $(3n^2 - 12mn + 6m^2) \div 3$

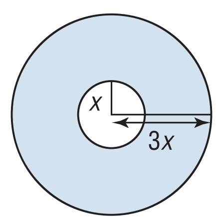
b) $\frac{-6rs - 16r - 4s}{-2}$

c) $\frac{10gh - 30g^2 - 15h}{5}$

d) $(12t^2 - 24ut - 48t) \div (-6)$

Take It Further

24. The area of a circle is given by the monomial πr^2.
Write, then simplify a polynomial for the shaded area in this diagram:



Reflect

How are multiplying and dividing a polynomial by a constant related?
Use examples to explain.